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Orbital angular momenta of quarks and gluons in the nucleon – model-dependent versus model-independent extractions –

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Abstract. We demonstrate that there exist two kinds of gauge-invariant decompositions of the nucleon spin, which are physically inequivalent. The quark and gluon orbital angular momenta (OAMs) appearing in one decomposition are basically the *canonical* OAMs, while the quark and gluon OAMs appearing in another decomposition are called the *dynamical* OAMs. It is shown that the dynamical OAMs of quarks and gluons in the nucleon can be related to definite high-energy deep-inelastic-scattering observables. On the other hand, we conjecture that the canonical OAMs of quarks and gluon in the nucleon are model-dependent quantities, which is meaningful only within a specific theoretical model of the nucleon.

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INTRODUCTION

The current status and homework of the nucleon spin problem can briefly be summarized as follows.. First, the quark polarization was fairly precisely determined to be around $1/3$. Second, gluon polarization is likely to be small, although with large uncertainties. What carries the remaining 2/3 of the nucleon spin, then ? Quark OAM ? Gluon spin ? Or gluon OAM ? To answer this question unambiguously, we cannot avoid to elucidate the following issues. What is a precise definition of each term of the decomposition? How can we extract individual term by means of direct measurements? Especially controversy here are the OAMs of quarks and gluons.

MODEL-INDEPENDENT EXTRACTION OF QUARK AND GLUON ORBITAL ANGULAR MOMENTA IN THE NUCLEON

As is widely known, there have been two popular decompositions of the nucleon spin. One is the Jaffe-Manohar decomposition [1], while the other is the Ji decomposition [2]. In these popular decompositions, only the intrinsic quark spin part is common, and the other parts are totally different. An apparent disadvantage of the Jaffe-Manohar decomposition is that each term is not separately gauge-invariant except for the quark spin part. On the other hand, each term of the Ji decomposition is separately gauge-invariant. Unfortunately, further gauge-invariant decomposition of J^g into its spin and

orbital parts is given up in this well-known decomposition. An especially important observation here is that, since the quark OAMs in the two decompositions are apparently different, one must necessarily conclude that the sum of the gluon spin and OAM in the Jaffe-Manohar decomposition does not coincide with the gluon total angular momentum in the Ji decomposition.

Some years ago, a new gauge-invariant decomposition of nucleon spin was proposed by Chen et al. [3],[4]. The basic idea is to decompose the gluon field $\boldsymbol{A}$ into two parts, the physical part $\boldsymbol{A}_{phys}$ and the pure-gauge part $\boldsymbol{A}_{pure}$. Imposing some additional conditions, i.e. what-they-call the generalized Coulomb gauge condition, Chen et al. arrived at the decomposition of the nucleon spin in the following form :

$$\begin{aligned}\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2}\boldsymbol{\Sigma}\,\psi\, d^3x + \int \psi^\dagger\, \boldsymbol{x}\times(\boldsymbol{p} - g\boldsymbol{A}_{pure})\,\psi\, d^3x \\ &+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys}\, d^3x + \int E^{aj}\,(\boldsymbol{x}\times\nabla)\,A^{aj}_{phys}\, d^3x \\ &= \boldsymbol{S}'^q + \boldsymbol{L}'^q + \boldsymbol{S}'^g + \boldsymbol{L}'^g. \end{aligned} \tag{1}$$

An interesting feature of this decomposition is that each term is separately gauge-invariant, while allowing the decomposition of the gluon total angular momentum into its spin and orbital parts. Chen et al's claim arose quite a controversy concerning the definition of quark and gluon OAMs [3] -[11]. In our opinion, the only way to settle the controversy is to clarify a concrete relationship between the proposed decompositions and direct observables. We believe that we have succeeded to reach this goal, step by step, through the recent three papers [9],[10],[11].

In the 1st paper [9], we have shown that the way of gauge-invariant decomposition of nucleon spin is not necessarily unique, and proposed yet another gauge-invariant decomposition given in the following form :

$$\boldsymbol{J}_{QCD} = \boldsymbol{S}^q + \boldsymbol{L}^q + \boldsymbol{S}^g + \boldsymbol{L}^g, \tag{2}$$

where

$$\boldsymbol{S}^q = \int \psi^\dagger \frac{1}{2}\boldsymbol{\Sigma}\,\psi\, d^3x, \tag{3}$$

$$\boldsymbol{L}^q = \int \psi\, \boldsymbol{x}\times(\boldsymbol{p} - g\boldsymbol{A})\,\psi\, d^3x, \tag{4}$$

$$\boldsymbol{S}^g = \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys}\, d^3x, \tag{5}$$

$$\boldsymbol{L}^g = \int E^{aj}\,(\boldsymbol{x}\times\nabla)\,A^{aj}_{phys}\, d^3x + \int \rho^a\,(\boldsymbol{x}\times\boldsymbol{A}^a_{phys})\, d^3x. \tag{6}$$

The characteristic features of our decomposition are as follows. First, the quark parts of this decomposition is common with the Ji decomposition. Second, the quark and gluon spin parts are common with the Chen decomposition. A crucial difference with the Chen decomposition appears in the orbital parts. That is, although the sums of the quark and gluon OAMs in the two decompositions are the same, i.e.

$$\boldsymbol{L}^q + \boldsymbol{L}^g = \boldsymbol{L}'^q + \boldsymbol{L}'^g, \tag{7}$$

each term is different in such a way that

$$\boldsymbol{L}^g - \boldsymbol{L}'^g \;=\; -\,(\boldsymbol{L}^q - \boldsymbol{L}'^q) \;=\; \int \rho^a \,(\boldsymbol{x} \times \boldsymbol{A}^a_{phys})\; d^3x. \tag{8}$$

The difference arises from the treatment of the 2nd term of Eq.(6), which is *solely* gauge-invariant. We call this term the *potential angular momentum* term, since the QED correspondent of this term is the OAM carried by the electromagnetic field (or potential), which appears in the famous Feynman paradox raised in his textbook of classical electrodynamics. We have included this term into the *gluon* OAM part, while Chen et al. included it into the *quark* OAM part.

In the 2nd paper [10], we made covariant extenstion of gauge-invariant decompositions of the nucleon spin. Covariant generalization of the decomposition has twofold advantages. First, it is essential to prove frame-independence of the decomposition. Second, it generalizes and unifies the nucleon spin decompositions in the market. Basically, we find two physically different decompostions. The decomposition (I) contains the well-known Ji decomposition, although it also allows gauge-invariant decomposition of gluon total angular momentum into its spin and OAM parts. The decomposition (II) contains three known decomposition, i.e. those of Bashinsky-Jaffe [12], of Chen et al. [3],[4], and of Jaffe-Manohar [1], as will be shown below.

The startingpoint of our general analysis is a decomposition of the full gauge field into its physical and pure-gauge parts, simiar to Chen et al. Here, we impose only the following general conditions only. The first is the pure-gauge condition for A^μ_{pure} :

$$F^{\mu\nu}_{pure} \;\equiv\; \partial^\mu A^\nu_{pure} - \partial^\nu A^\mu_{pure} - i\,g\,[A^\mu_{pure}, A^\nu_{pure}] \;=\; 0, \tag{9}$$

while the second is the gauge transformation properties for these two components :

$$A^\mu_{phys}(x) \;\;\to\;\; U(x)\, A^\mu_{phys}(x)\, U^{-1}(x), \tag{10}$$

$$A^\mu_{pure}(x) \;\;\to\;\; U(x) \left(A^\mu_{pure}(x) - \frac{i}{g}\,\partial^\mu \right) U^{-1}(x). \tag{11}$$

Actually, these condition alone are not enough to fix gauge uniquely. However, the point of our analysis is that we can *postpone* a complete gauge fixing until later stage, while accomplishing a gauge-invariant decomposition of $M^{\mu\nu\lambda}$ based on the above conditions only.

We start with the decomposition (II) given as

$$\begin{aligned} M^{\mu\nu\lambda}_{QCD} \;&=\; M'^{\mu\nu\lambda}_{q-spin} \,+\, M'^{\mu\nu\lambda}_{q-OAM} \,+\, M'^{\mu\nu\lambda}_{g-spin} \,+\, M'^{\mu\nu\lambda}_{g-OAM} \\ &+\;\; \text{boost} \,+\, \text{total divergence}, \end{aligned} \tag{12}$$

with

$$M'^{\mu\nu\lambda}_{q-spin} \;=\; \frac{1}{2}\,\epsilon^{\mu\nu\lambda\sigma}\,\bar{\psi}\,\gamma_\sigma\,\gamma_5\,\psi, \tag{13}$$

$$M'^{\mu\nu\lambda}_{q-OAM} \;=\; \bar{\psi}\,\gamma^\mu\,(\,x^\nu\, i D^\lambda_{pure} \,-\, x^\lambda\, i D^\nu_{pure}\,)\,\psi, \tag{14}$$

$$M'^{\mu\nu\lambda}_{g-spin} \;=\; 2\,\mathrm{Tr}\,\{\,F^{\mu\lambda}\,A^\nu_{phys} \,-\, F^{\mu\nu}\,A^\lambda_{phys}\,\}, \tag{15}$$

$$M'^{\mu\nu\lambda}_{q-OAM} \;=\; 2\,\mathrm{Tr}\,\{\,F^{\mu\alpha}\,(\,x^\nu\, D^\lambda_{pure} \,-\, x^\lambda\, D^\nu_{pure}\,)\,A^{phys}_\alpha\,\}. \tag{16}$$

At first sight, this decomposition appears a covariant generalization of Chen et al's decomposition. However, a crucial difference is that we have not yet fixed gauge explicitly. Owing to this general nature, our decomposition (II) reduces to any ones of Bashinsky-Jaffe [12], of Chen et al. [3],[4], and of Jaffe-Manohar [1], after an appropriate gauge-fixing in a suitable Lorentz frame, which dictates that these 3 decompositions are all *gauge-equivalent* ! They are not recommendable decompositions, however, because the quark and gluon OAMs in those do not correspond to *known* experimental observables.

Our recommendable is the decomposition (I) given as

$$\begin{aligned} M^{\mu\nu\lambda} &= M^{\mu\nu\lambda}_{q-spin} + M^{\mu\nu\lambda}_{q-OAM} + M^{\mu\nu\lambda}_{g-spin} + M^{\mu\nu\lambda}_{g-OAM} \\ &+ \text{ boost } + \text{ total divergence}, \end{aligned} \tag{17}$$

with

$$M^{\mu\nu\lambda}_{q-spin} = M'^{\mu\nu\lambda}_{q-spin}, \tag{18}$$

$$M^{\mu\nu\lambda}_{q-OAM} = \bar{\psi}\gamma^\mu (x^\nu iD^\lambda - x^\lambda iD^\nu)\psi \neq M'^{\mu\nu\lambda}_{q-OAM}, \tag{19}$$

$$M^{\mu\nu\lambda}_{g-spin} = M'^{\mu\nu\lambda}_{g-spin}, \tag{20}$$

$$M^{\mu\nu\lambda}_{g-OAM} = M'^{\mu\nu\lambda}_{g-OAM} + 2\,\mathrm{Tr}[(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda_{phys} - x^\lambda A^\nu_{phys})]. \tag{21}$$

It differs from the decomposition (II) in the orbitals parts. The quark OAM part contains full covariant derivative contrary to the decomposition (II). Correspondingly, the gluon OAM part is also different. It contains a covariant generalization of the potential angular momentum term. A superiority of this decomposition is that the quark and gluon OAMs in this decomposition can be related to experimental observables !

Before demonstrating the superiority of the decomposition (I), it may be instructive to confirm the fact that the difference between the quark and gluon OAMs in the two decompositions is *nothing spurious*, i.e. it is *physical*. This can, for instance, be convinced from an explicit model analysis by Burkardt and BC [13]. Using simple toy models, i.e. scalar diquark model and QED and QCD to order α, they compared the fermion OAM obtained from the Jaffe-Manohar decomposition [1] and from the Ji decomposition [2]. In our terminology, these two fermion OAMs are nothing but the *canonical* OAM and the *dynamical* OAM. Their findings are as follows. The two decompositions give the same fermion OAMs in scalar diquark model, but they do not in QED (gauge theory). The x-distributions of fermion OAMs are different even in scalar diquark model. In QED and QCD at order α, two kinds of OAMs are definitely different. Although their analysis is highly model-dependent, an important lesson to learn is that one should clearly distinguish two kinds of OAMs, i.e. the canonical OAM (including its nontrivial gauge-invariant extension due to Chen et al.) and the dynamical OAM, the difference of which is nothing spurious, i.e. physical.

Now we shall explain why we recommend the decomposition (I). The keys are the following identities, which holds for the quark and gluon OAM operators of our

decomposition (I). For the quark part, we have

$$\begin{aligned} L_q &= J_q - \frac{1}{2}\Delta q \\ &= \frac{1}{2}\int_{-1}^{1} x\,[H^q(x,0,0) + E^q(x,0,0)]\,dx - \frac{1}{2}\int_{-1}^{1}\Delta q(x)\,dx \\ &= \langle p\uparrow|M^{012}_{q-OAM}|p\uparrow\rangle, \end{aligned} \tag{22}$$

with

$$M^{012}_{q-OAM} = \bar{\psi}\left(x\times\frac{1}{i}D\right)^3\psi \;\neq\; \begin{cases} \bar{\psi}\left(x\times\frac{1}{i}\nabla\right)^3\psi \\ \bar{\psi}\left(x\times\frac{1}{i}D_{pure}\right)^3\psi. \end{cases} \tag{23}$$

We find that the proton matrix element of our quark OAM operator coincides with the difference between the 2nd moment of GPD $H+E$ and the 1st moment of the longitudinally polarized distribution of quarks. What should be emphasized here is that full covariant derivative appears, not a simple derivative operator nor its nontrivial gauge-invariant extension. In other words, the quark OAM extracted from the combined analysis of GPD and polarized PDF is *dynamical* OAM (or *mechanical* OAM) not *canonical* OAM. This fact is nothing different from Ji's claim.

Similarly, for the gluon part, we find that the difference between the 2nd moment of gluon GPD $H+E$ and the 1st moment of polarized gluon distribution coincides with the proton matrix element of our gluon OAM operator given as follows :

$$\begin{aligned} L_g &= J_g - \Delta G \\ &= \frac{1}{2}\int_{-1}^{1} x\,[H^g(x,0,0) + E^g(x,0,0)]\,dx - \int_{-1}^{1}\Delta g(x)\,dx \\ &= \langle p\uparrow|M^{012}_{g-OAM}|p\uparrow\rangle, \end{aligned} \tag{24}$$

with

$$\begin{aligned} M^{012}_{g-OAM} &= 2\,\mathrm{Tr}\,[E^j\,(x\times D_{pure})^3 A^{phys}_j] && : \text{ canonical OAM} \\ &+ 2\,\mathrm{Tr}\,[\rho\,(x\times A_{phys})^3] && : \text{ potential OAM term.} \end{aligned} \tag{25}$$

Namely, the gluon OAM extracted from the combined analysis of GPD and polarized PDF contains *potential* OAM term, in addition to *canonical* OAM. It would be legitimate to call the whole part the gluon *dynamical* OAM.

A natural next question is why the dynamical OAM can be observed ? An answer can be found in the famous textbook of quantum mechanics by J.J. Sakurai [14]. There he discusses a motion of a charged particle in static electric and magnetic field. It is emphasized that, under the electromagnetic potential, the dynamical (or mechanical) momentum $\mathbf{\Pi} \equiv m\frac{d\boldsymbol{x}}{dt}$, defined as a product of mass and particle velocity, is given by $\mathbf{\Pi} \equiv \boldsymbol{p} - e\boldsymbol{A}$, which is different from the canonical momentum $\boldsymbol{p}$. Furthermore, what appears in the quantum version of Newton's equation of motion is a dynamical momentum $\mathbf{\Pi}$ not a canonical one $\boldsymbol{p}$. "Equivalence principle" of Einstein dictates that

the *flow of mass* can in principle be detected by using gravitational force as a probe. Naturally, the gravitational force is too weak to be used as a probe of mass flow accompanying the motion of microscopic particle. However, remember that the 2nd moments of unpolarized GPDs are also called the gravito-electric and gravito-magnetic form factors. The fact that the dynamical OAM as well as dynamical linear momentum (fraction) can be extracted from GPD analysis is therefore not a mere accident !

Our final comment is on the *quantum loop effects*, which have not been considered in our analysis so far. Here, it may be instructive to restart the whole argument with a general reasoning deduced from the widely-accepted fact as follows. By now, no one would object to the fact that the nucleon spin can be decomposed into the total angular momenta of quarks and gluons in a gauge-invariant way by means of GPD measurements. Since the quark polarization is gauge-invariant and measurable through the polarized DIS measurements, the quark OAM defined as a difference between the total quark angular momentum and the longitudinal quark polarization is clearly gauge-invariant and observable. The gluon part is a little more subtle. If ΔG is really gauge-invariant and measurable, the gluon OAM defined as a difference between the total gluon OAM and the gluon polarization should be gauge-invariant and observable. Therefore, a key question is 'Is ΔG really a gauge-invariant quantity or not ?"

This is a very delicate question. In fact, it was often claimed that ΔG has its meaning only in the light-cone gauge and infinite-momentum frame. More specifically, in an influential paper [15], Hoodbhoy, Ji, and Lu concluded that ΔG evolves differently in the Feynman gauge and the LC gauge. However, the gluon spin operator used in their Feynman gauge calculation is not gauge-invariant and delicately different from our gauge-invariant gluon-spin operator. The question is how to introduce this difference into the Feynman rule of evaluating 1-loop anomalous dimension of the quark and gluon spin operators. This problem was attacked and solved in our 3rd paper [11]. We find that the calculation in the Feynman gauge (as well as in any covariant gauge including the Landau gauge) reproduces the answer obtained in the LC gauge, which is also the answer obtained in the famous Altarelli-Parisi method.

Our finding is important also from another context. So far, a direct check of the answer of Altarelli-Pasiri method for the evolution of ΔG within the operator-product-expansion (OPE) framework was limited to the LC gauge only, because it has been believed that there is no gauge-invariant definition of gluon spin in the OPE framework. This is the reason why the question of gauge-invariance of ΔG has been left in unclear status for a long time ! Now we can definitely say that the gluon spin operator appearing in our nucleon spin decomposition (although nonlocal) certainly provides us with a completely satisfactory operator definition of gluon spin with full gauge invariance, which has been searched for nearly 40 years.

MODEL-DEPENDENT INSIGHTS INTO THE QUARK AND GLUON ORBITAL ANGULAR MOMENTA IN THE NUCLEON

In the previous section, we have emphasized the existence of two kinds of orbital angular momenta, i.e. the *canonical* and *dynamical* OAMs. We argued that the *dynamical* OAM can be observed through the combined analyses of unpolarized GPDs and longitudinally

polarized PDFs. Is there any possibility to extract *canonical* OAM by means of direct measurements ? If this is possible, it means that we can isolate the very *correspondent* of *potential angular momentum* appearing in the Feynman paradox. Unfortunately, we are a little pessimistic about this possibility by the reason explained below.

To explain it, we first recall a model-dependent sum rule for the quark OAM in the nucleon advocated by Avakian et al. [16]. They showed that, within the framework of the MIT bag model (and also in scalar diquark model), a certain weighted-integral of a T-even and chiral-odd TMD distribution $h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$, called the pretzolocity, reduces to the OAM of quarks in the nucleon as

$$\langle L_3^q \rangle \ = \ - \int dx \int d^2 \boldsymbol{k}_\perp \, \frac{\boldsymbol{k}_\perp^2}{2M} \, h_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2). \tag{26}$$

Taking $Q = u + d$, we have

$$\langle L_3^Q \rangle \ = \ \frac{2}{3} \, P_P, \tag{27}$$

with

$$P_S \ = \ \int_0^R [\, f(r)]^2 \, r^2 \, dr, \quad P_P \ = \ \int_0^R [g(r)]^2 \, r^2 \, dr. \tag{28}$$

Here, $f(r)$ and $g(r)$ are the radial parts of the upper and lower components of the ground state wave function of the MIT bag model. The complete expression of the nucleon spin decomposition is also very simple in the MIT bag model :

$$\langle J_3 \rangle \ \ = \ \ \langle L_3^Q \rangle \, + \, \frac{1}{2} \langle \Sigma_3^Q \rangle \ = \ \frac{2}{3} \, P_P \, + \, \frac{1}{2} \left(P_S - \frac{1}{3} \, P_P \right) \ = \ \frac{1}{2}. \tag{29}$$

Unfortunately, MIT bag model is not a realistic model of the nucleon, which is a bound state of nearly zero-mass quarks ! Important physics like chiral symmetry is not properly taken into account. More serious would be the neglect of gluon degrees of freedom, which are widely believed to carry sizable amount of nucleon momentum fraction. In any case, the above simple relation (26) obtained in the MIT bag model must be taken with care. The point is that the probability P_P related to the quark OAM is a highly model-dependent quantity.

A deeper meaning of our statement above may be understood by considering much simpler composite system, i.e. the deuteron. It is known that, in the simplest approximation, the magnetic moment of the deuteron is given by the formula :

$$\mu_d \ \ = \ \ \mu_p \, + \, \mu_n \, - \, \frac{3}{2} \, P_D \left(\mu_p + \mu_n - \frac{1}{2} \right). \tag{30}$$

Here, μ_p and μ_n are the magnetic moments of the proton and neutron, while P_D is the so-called D-state probability of the deuteron. The above formula indicates that, by measuring the magnetic moment, one can extract the D-state probability of the deuteron. This D-state probability also gives the measure of OAM contents of the deuteron, as is clear from the following angular momentum decomposition of the deuteron spin.

$$\langle J_3 \rangle \ \ = \ \ \langle L_3 \rangle \, + \, \langle S_3 \rangle \ = \ \frac{3}{2} \, P_D \, + \, \left(P_S - \frac{1}{2} \, P_D \right) \ = \ 1. \tag{31}$$

However, it is a well-known fact that the D-state probability of the deuteron, which is thought to be an analogous object to the probability P_P of the MIT bag model, is not a direct observable [17],[18]. We emphasize that the OAM, which we discuss above corresponds to an expectation value of *canonical* OAM operator between some Fock-state eigenvectors, which has a definite meaning only within a specific theoretical model.

SUMMARY AND CONCLUSION

We have discussed the OAM of composite particles, with particular emphasis upon the existence of two kinds of OAM, i.e. canonical OAM and dynamical OAM as well as canonical momentum and dynamical momentum. The canonical momentum is certainly a fundamental building block in the theoretical framework of quantum mechanics and quantum field theory. However, whether it corresponds to an observable is a different thing ! In contrast, we have shown that the dynamical OAM of quarks and gluons in the nucleon can in principle be extracted model-independently from combined analysis of the GPD measurements and the polarized DIS measurements. This means that we now have a satisfactory theoretical basis toward a complete decomposition of the nucleon spin, which is a strongly-coupled relativistic bound system of quarks and gluons.

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